A Linearized Model of C-41 Material's H&D Curves with Orange Mask

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1 Notation

I use $\overrightarrow{V}$ to denote a vector. Vectors used in this document have one component for each wavelength range under consideration. These wavelength ranges typically comprise a red, and green and a blue light range, aligned with typical film sensitization and human eye sight.

Matrices are denoted as $\underset{\sim}{M}$ and are typically 3x3 matrices representing the linear relationship between one vector and another. The quantity M_{ij} denotes the matrix element in the i^{th}row and j^{th} column. The inner product between $\underset{\sim}{M}$ and $\overrightarrow{V}$ yielding result vector $\overrightarrow{R}$ is denoted as $\underset{\sim}{M} \bullet \overrightarrow{V} = \overrightarrow{R}$.

The quantity δ_{ij} is the Kronecker delta, which equals 1 for $i = j$ and 0 for $i \neq j$.

2 Modeling of linear section of H&D curve

Typical H&D curves for the three color layers of C-41 color negative material follow roughly the same pattern as traditional B&W materials: beginning at low exposures, after some flat section with constant and (ideally) low density the H&D curve enters a curved "toe section", after this comes a part with nearly linear appearance, followed by a "shoulder section" with decreasing steepness.

In typical pictorial scenes, which C-41 film is optimized for, most image data happens within this straight line section. The area of beginning flat line and toe section are turned into featureless black areas in the final image, whereas the shoulder region is typically turned into near featureless white area. Color correction measures will therefore focus on this straight line section of the H&D curve.

Modeling of Color Reproduction Errors

H&D curves for color negative film are typically shown as a set of three independent curves, one for each color channel. This representation implies, that exposure with one light wavelength will only affect its own layer. There are, however, two reasons, why this may not be the case with realistic emulsions: a color layer may be sensitive to wavelengths outside its intended sensitivity range, and the dyes formed during color development may have absorption outside its intended wavelength range.

Both effects will give "wrong" colors, and the error can be approximated by the same formula. If we write exposure as vector $\overrightarrow{E}$ and density as vector $\overrightarrow{D}$, with each element representing that property for one wavelength range (red, green, blue), the straight line segment can be approximated as $\overrightarrow{D} = \underset{\sim}{\mathit{\Gamma}} \bullet \overrightarrow{E} + \overrightarrow{D_0}$, where $\underset{\sim}{\mathit{\Gamma}}$ is a 3x3 matrix and $\overrightarrow{D_0}$ represents base density. Element $\mathit{\Gamma}_{ij}$describes the effect of exposure in wavelength range j on density in wavelength range i.

In an ideal world, $\mathit{\Gamma}_{ij}$would be 0 for $i \neq j$. Depending on developer composition it is possible to force all γ_{ii} to have the same value γ, in which case we can write $\Gamma_{ij} = \gamma * \delta_{ij}$. The term γ would then denote the common slope of three distinct and independent characteristic curves. Differences between these γ_{ii} would be seen as color crossover, i.e. different slopes of individual color channels.

3 Orange Mask

In order to improve color reproduction, special types of dye couplers has been used. These couplers have some color of their own, and after coupling with oxidized developer molecule, the original dye is destroyed and the actual image dye is formed. Therefore we get an extra term in this formula from above: $\overrightarrow{D} = \underset{\sim}{\mathit{\Gamma}} \bullet \overrightarrow{E} - \underset{\sim}{M} \bullet \vec{E} + \overrightarrow{D_0} = (\underset{\sim}{\Gamma} - \underset{\sim}{M}) \bullet \overrightarrow{E} + \overrightarrow{D_0}$. The term $\overrightarrow{D_0}$ will have a different, and typically substantially higher value than in mask less film.

The matrix $\underset{\sim}{\mathit{\Gamma}}$ is still the 3x3 matrix representing creation of image dye in exposed regions, whereas the matrix $\underset{\sim}{M}$ represents the orange mask which yields reduced density with increasing exposure and development. While it appears to be exceedingly difficult to find dye couplers with $\mathit{\Gamma}_{ij} = \delta_{ij} * \gamma$, it appears to be technically feasible to find special coupler molecules, for which $\mathit{\Gamma}_{ij} - M_{ij} \sim \delta_{ij} * \gamma$. These special molecules, of which one suitable type is needed for each color channel, are the ones used in modern, orange masked film.